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Use of Non-commercial Software in Mathematics

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ABSTRACT

This work describes the use of non-commercial software in terms of innovation of traditional educational process in mathematics in higher education in Serbia which is largely based on the concepts in the last century, with a very low teaching adjustment which follows individual characteristics and abilities of students, so today it gives very limited results in efficiency and in study duration. The idea behind the non-commercial software is to simplify the understanding of basic terms and concepts for students, which are studied in subjects related to mathematics, and also to reduce the cost of higher education institutions. High price of commercial software can often make them inaccessible to many of our institutions and individuals. Non-commercial educational mathematical software are designed for innovative, interactive and dynamic teaching in various areas of mathematics that is the basis for development of logical thinking necessary for further education. Their use would have a significant role in distance education that involves the physical distance between teachers and students as well as benefits for students using it in the process of visual mastering of mathematical material. This work suggests a simple and free model in mathematics that includes information technology. Such model can be a good recommendation for mathematics in universities and other higher education institutions that have a similar curriculum in mathematical subjects. In this work the emphasis is on GeoGebra Octave and Maxim non-commercial programs which in mathematical sense join geometry, algebra and analysis. There is also an explanation how to use this software in mathematics. The work is written with the purpose of popularizing free software GeoGebra, Octave and Maxima, and all with the idea of improving the quality of mathematics in the higher education institutions.

INTRODUCTION

"In the mathematics classes there is a need for harmonically connection of facts, skills, conceptual structures, methods and general strategies in solving problems. It must be admitted that this is not an easy task. There is also a rooted aversion to

mathematics as "difficult subject". Question: "Why is mathematics difficult to learn?" is a challenge for the designer of computer software also. It is necessary to thoroughly examine all relevant factors for the acquisition of mathematical knowledge. The research of human learning can significantly contribute of making the learning mathematics more efficient. The main problem of teaching mathematics is that it involves working at a high level of abstraction. This raises the question of how to teach mathematics to develop abstract thinking. It is known that children replace intuitive models with the concrete (e.g., their perceptions about the space are adapted to the physical world), but on the other hand, if they want to develop their knowledge (e.g. in geometry) then they need to locate their thoughts on much abstract level then the realistic. One of the possible teaching strategies is to overcome the symbolism and to break into the "realistic mathematics", and the second is to enable teachers that in dialogue with students, provoking them with the wrong concepts, create the conditions where student is changing the way of thinking. In any case, when teaching mathematics there must be an information feedback compared to each student activity. In modern mathematics there are more and more presents of the new information technology which contributes better performance of mathematical education. When asked how IT solutions can contribute the mathematics knowledge increasing, the answer is, first of all, to use computer software when teaching and learning mathematics which optimally meets the objectives of mathematics education. In this way we achieve individualization of teaching, which is otherwise difficult to achieve in classical mathematics. A great impact is that graduate students have the ability to design and perform an experiment, and also to analyze and interpret data, also they have the ability to identify, formulate and solve problems within their profession. Given that mathematical subjects are basis for developing logical thinking it is of great importance to improve the teaching of mathematics in every way. Proposition for updating the teaching of mathematics may be: e-learning, applications that allow the student to attend classes; software tools in aim to make teaching innovative, interactive and dynamic and to link the programs with platforms for distance education as a support to the traditional educational process are essential for the learning process.

FREE SOFTWARE

Anyone can use, copy, distribute, study, change and improve free software whether free or for a price. In order to be free software requires access to source code. The software is free, if users have the right (freedom) to run the program for any reason, testing the programs and adapting it to their needs, access to source code, redistribution of copies. Regardless of whether users pay to get copies of GNU (an acronym "GNU's Not Unix) software, or they get it for free, they will always have freedom to copy or modify the software, even to sell copies. Free software may be available for commercial use. Such programs are free commercial programs. The right (freedom) to improve the programs, and release the improvements to the public, seems the community benefits from. Access to the source code is a precondition for this right. Open software (open source software) refers to software

whose source code is available within the "open source" licenses to all users who can edit, modify and improve its content. This means that the "open source" software comes with the whole source code in some programming language, so the program itself can be changed, which is not the case with paid software. In 1998th, a group of individuals presented the term "open source" to replace the term "free software". This term marked the beginning of the software that could download for free from the Internet, examine and modify its source code. Open source represents good quality, high reliability, more flexibility and lower costs. The purpose of open source software is to make programs understandable and accessible, for further, more efficient progress. Open software movement is supported by a large number of programmers, scientists and other computer users who support unlimited access to the source code of the program. Supporters of open source software believe that their mode is superior than the mode of closed software makers, in other words, that with common, joint and unlimited work it is possible to make the program work better than the ones made in closed projects. According to supporters of "open source" software, it is the future of computing, because the unselfish sharing of knowledge could offer all the available knowledge to existing and some potential developers.

MATHEMATICAL SOFTWARE TOOLS

Mathematical software tools have an important role in teaching and learning. The primary use of computer tools for mathematical purposes is a quality representation and verification of results. Mathematical software tools are designed for innovative, interactive and dynamic teaching in various areas in mathematics. There are a number of different software packages designed for working with mathematical content, such as Mathematica, Maple, Scientific WorkPlace, manipulate Math with Java, Derive and Calculus T / L II, Algebra, GeoGebra and others. Most of them allows symbolic, numerical and graphical operations, so in these way computers graphic capabilities ensures mathematics to be seen, algebraic part of the software ensures that mathematics can bee done, and using the expressiveness of programming language mathematics can bee created. Fine software packages designed for teaching and learning of mathematics have the following characteristics: user has complete control while using the technology; to encourage and motivate the research and testing; some options are available that would not be without the use of technology, clear and intuitive interface; easy entry of input data ; immediate verification of output data (instead of "check your answers at the end of the book") with full explanation, flexible and intuitive manipulation using mathematical forms, graphics and tabular values; two-dimensional or three-dimensional view of input and output parameters, repetition of each step of the transaction for individual training; software is fully in function of mathematics: the user accepts mathematical concepts and skills with better commitment and higher level of understanding.

MATHEMATICS APPLETS

Mathematics Applets are Java applications that run inside Web pages. Java applets allow the user to operate with geometric objects and algebraic calculations not necessary using any special mathematical programs. Java applets are interactive, intuitive and simple applications, which can be excellent for teaching and learning mathematics. To view this site it is necessary that user has Java installed on his computer, that is otherwise widely used on the Internet. Software package "Manipula Math with Java" is designed for high school students, students of colleges and universities, as well as all others interested in mathematics. The software contains interactive programs easy to use, as well as many animations that help to achieve the full meaning of mathematical ideas. Using this software students "can see" the process ahead, so they can make relevant parameters change, they can observe the reverse reaction of a change in the process, so in that way they somehow become part of the process. Student can repeat the Figure any number of times, until he experience it in right way, which makes the process better to understand and to remember.

CATM and CAS tools

CATM (Computer-assisted tools for mathematics) are tools that help user in solving some mathematical problems. The term CATM is used to indicate the software that performs numerical calculations, as well as software that performs symbolic calculations. The term CAS as shortcut for "Computer Algebra System" is limited to software that performs symbolic calculations. Accordingly, CAS tools are subset of CATM tools. There is a wide range of commercial CATM tools that are not for free, but they are well documented, they are supported by various operating systems, and also incredibly powerful and flexible (Mathematica, Maple, Matlab, ...). Unfortunately, each of the commercial products are very expensive and, due to its cost, are inaccessible for many of our institutions and individuals. Matlab is mainly intended for engineers, especially to those working with complex mathematical calculations. It is primarily designed for numerical calculations. Mathematica is one of the first tools in CATM type that also supported symbolic calculation. Therefore, the user can get a symbolic formula as an answer to the question, instead of pure "numeric" values. Maple is another popular CATM that can perform numeric and symbolic calculations.

FOSS-CATM

FOSS (Free/Open Source Software) represents computerized tools for various fields of mathematics. Benefits for students: This way they can learn through experimentation, from any place they want or from home, without the constant presence of teachers. Benefits for teachers: they can give assignments for practice, projects, homework, and expect from all students to do so, they can demonstrate important concepts using visualization tools. By using this software teacher can increase the quality of teaching. Benefits for institutions: there is no need to set aside money to buy this software, nor they can have a problem with illegal use of software. They can use the software for their own needs, in a way that standard of

teaching certainly raises to a higher level. There is a large number of mathematical tools designed for free use. The following software packages are FOSS-CATM type: Cocoa, GPL, Scilab, Scicos, GNU / Octave, GNU GPL, Maxima, GPL, GeoGebra, GNU GPL, Axiom, modified BSD license; Bergman, GPL-style license; Cadabra, GPL; FriCAS, modified BSD license; Mathomatic, LGPL; OpenAxiom, modified BSD license. Common feature of the above CATM tools is that they present very important products intended for serious applications.

Those mathematical tools have multiple supports from different operating systems. All open source software packages are running under the Linux operating system, and only a small number of packages does no have built-in support for operating systems: Windows, Mac OS X and BSD.

Scilab (FOSS CATM) is perhaps the best clone of a commercial software package MATLAB, designed for numeric and symbolic mathematics.

Users are provided by good support in various forms: mailing lists, Usenet groups, Web sites. Scilab is constantly updated and maintained. Files like *.m work on a platform of Scilab, and thanks to the command for translation (m2sci) Scilab is interpreter. The user enters a command (or more commands together in one script-file), and starts its execution. There are number of functions designed for visualization of mathematical content in 2D or 3D space, as well as for animation of mathematical content. Scilab allows export graphics files in LaTeX, for further work.

Scicos (Simulink of Matlab) that comes for free with Scilab, create block diagrams, models and simulates the dynamics of different hybrid dynamic systems. CoCoA (Computations in Commutative Algebra) is free open source software system that can be applied in fields such as polynomials, vector and linear algebra.

Software platform can be installed in different operating systems. Cocoa is specialized in calculation of polynomials (multiplication, division, separation factors and the like.), also in solving systems of homogeneous and inhomogeneous linear equations, in solving wide range of vector and matrix algebra tasks, etc..

With command type of interactive interface there is well done help system that is available for users. Axiom is a software designed for research purposes and also as a support in traditional education.

It is useful for symbolic calculations, mathematical research and development of mathematical algorithms. Axiom is a system with an interactive environment, with its own programming language and a large set of libraries for numeric, symbolic and algebraic calculations. High programming language contains commands that express abstract and complex mathematical concepts. Axiom is a competitor to the program package Mathematica, and it best works under the Linux operating system. Axiom can work as a calculator, support the work with standard features, with fixed number of digits, and it can also work with very complex numeric and symbolic calculations. Gnuplot is a free tool for two-dimensional or three-dimensional plot based on data and functions. It supports multitude of output formats, including LaTeX. It is very powerful, and can be used as part of Octave, like plotting machine.

GEOGEBRA

Markus Hohenwarter is the creator of free software for open-source dynamic mathematics GeoGebra, designed for mathematics teaching from high school to university level. The software is available at www.geogebra.com, the application is available on all platforms that support Java Virtual machine. The software is designed for educational purposes especially for math courses. GeoGebra is dynamic geometry software as it supports the construction of points, lines and all conic sections, which can be dynamically changed. On the other hand, GeoGebra provides some typical functions of computer algebra system, such as finding the significant points of the function (zeros, extreme points, inflection points), directly determine equations and coordinates, and calculation of abstracts and integrals of given functions. Therefore, GeoGebra is good choice for multiple representations of mathematical objects. Appliance would be most useful for students in solving mathematical problems in class or elsewhere. On the other hand, GeoGebra offers significant opportunities for teachers to create interactive educational materials that may be available on the Internet. GeoGebra encourages teachers to use technology in teaching, in fields like: visualization of mathematics; "research" in mathematics, interactive lessons over site, or remotely, various forms of application of mathematics, and so on. Also, experienced teachers can convert such dynamic, virtual worksheets into interactive exercise, using GeoGebra JavaScript interface and parameters for the applet setting. As of result of that is tremendous flexibility achieving, creating exercises with automatic feedback, or, exercises that may be randomly generated by each student. GeoGebra provide for teachers new possibilities for creating interactive, stimulating online learning environment that will encourage many of them to distribute them as free, divisible material on the Internet. Computer algebra systems (CAS) (e.g., Mathematica, Maple, etc.) and dynamic geometry software (such as Geometer's Sketchpad, Cabri Geometry, etc.) represent powerful technology tools designed for mathematics teaching. As a result of numerous studies it indicates that these software packages can be used to encourage research, experimentation and visualization in traditional mathematics teaching. However, some research suggests that for most teachers only supply of technology can be a problem for the successful integration of technology into teaching. In that sense, recommended solution for application of technology in teaching higher mathematics is certainly a software package GeoGebra. The advantages of using GeoGebra are: Compared with graphing calculator, GeoGebra is a more "friendly" (eng. user-friendly). GeoGebra has easy to use interface (eng. easy-to-use interface), multiple language support for menus, commands and help. Students can easily customize crested solutions by changing the interface (for example, font size, language, graphics quality, color selection, coordinate, thickness and line style, etc.).. GeoGebra encourages development of student projects in mathematics, multiple representations, as well as experimental and research-guided learning (eng. guided discovery learning). GeoGebra is created to help students to understand mathematics better. Students can easily manipulate variables, simply by dragging the "free" objects on the surface of drawing, or by using the slider. Students, using the technique of manipulation of free objects, generate changes, and

then observed how this affects the doings of dependent objects. In this way, students were given the opportunity to solve and observe some problems using dynamic study of mathematical relationships. GeoGebra provides good opportunities for cooperative learning. Cooperative learning represents good selected mode for many mathematical fields. Classical lectures should be replaced with a theme-oriented interactive classroom. The primary teacher's role is not a lecture, explanation, or some other kind of trying to "transfer" mathematical knowledge, but to create conditions that will enable students to "build" the necessary mental constructions quickly. In this sense, GeoGebra provides multiple opportunities for cooperative learning, like, working in small groups or whole class interactive teaching, or individual/group student presentations. Algebraic part of interface using command line, allows users to generate new objects or modify existing ones. Created worksheets can easily be published as Web pages. GeoGebra encourages teachers to use technology in teaching, in fields like: visualization of mathematics, "research" in mathematics, interactive lessons by web site or remotely, various forms of mathematics use, and so on. Popularity of GeoGebra is rapidly growing among teachers around the world, precisely because the dynamic, mathematical software package is easy to use and which also combines many aspects of various software packages. GeoGebra due to its open source nature has is supported by huge program community that continues to develop and improve it on a daily basis. GeoGebra is an easy way for learning mathematics. Students may explore the functions, draw geometric objects and calculate some algebraic tasks. Thanks to an interactive website users from different countries can work together on some math problems. GeoGebra is localized around the world and that is why communication and language comprehension is not an issue. GeoGebra has the ability to export the construction in HTML and as such can be imported into Moodle. Figure 1 shows an example of GeoGebra incorporated in Moodle.

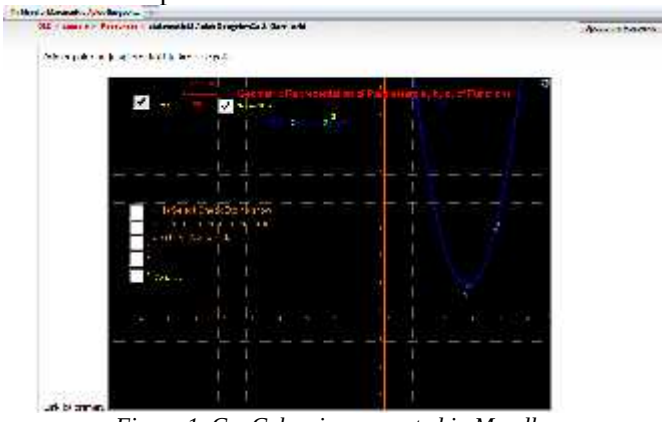

Figure 1. GeoGebra incorporated in Moodle

Disadvantages of using GeoGebra are:

Students with no previous programming experience will be hard to manage with inputting algebraic commands via interface input field. Although it is not difficult to learn the basic commands of GeoGebra, some students may feel uncomfortable or completely lost.

Also, some methodological approaches (for example, independent "research" and experimentation) may be inadequate for many students. Without proper guidance and explanations by the teacher, many students who did not understand the material on a regular teaching and do not have the necessary knowledge for such work, will make unarticulated, ineffective and meaningless actions. In technical terms, GeoGebra has no built- in support for animation and 3D display. Technical upgrade with GeoGebra module for animation and 3D display, should become an important element of future versions of this package. Future enlargement of GeoGebra software will certainly include major symbolic features of the current CAS, which will increase the possible complex application of GeoGebra in mathematical analysis field. A small number of scientific studies on the impact of GeoGebra in teaching and learning of mathematics. In other words, this tool extends the concept of dynamic geometry on the fields of algebra and calculus. Students can simultaneously use the CAS (Computer Algebra System) and an interactive geometry system, thereby students are significantly increasing their cognitive abilities.

OCTAVE and MAXIMA

GNU Octave, emerged as a reaction to FORTRAN, is computer language primarily designed for numerical calculating. It is partly compatible with MATLAB, and compatibility improves from version to version. Octave has the tools for solving problems in linear algebra, finding roots of nonlinear equations, polynomial operations and integration of functions and differential equations. Although the name of a free program Octave reminds of music, Program is named after Professor Octave Levenspiel, who was a one of the authors of the program professor. J. Rawlings and J. Ekerdt wrote the first version of the program. In the purpose of comfortable work, this program is using its own upgrade QtOctave, because Octave has no graphical interface. The instruction of Octave and its upgrade QtOctave installation can be downloaded from the internet for free. Figure 2. shows the appearance of the main window, which is obtained by running the QtOctave software.

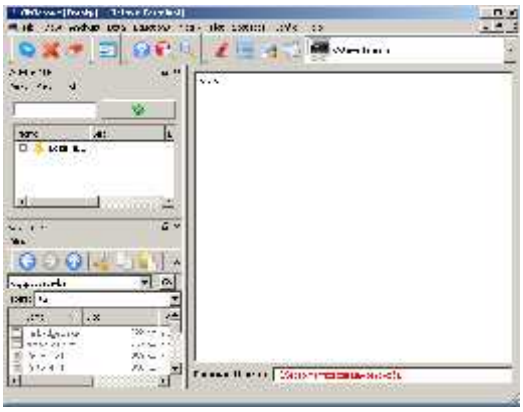

Figure 2. QtOctave - the main window

MAXIMA is a computer algebra system, which allows symbolic solving of mathematical problems. Because working with MAXIMA requires the command prompt writing, we will use WXMAXIMA, the program that presents a graphical interface for software MAXIMA. Figure 3. shows the appearance of the main window, which is obtained by running the WXMAXIMA software.

Figure 3. WXMAXIMA - the main window

These programs are free to download from the Internet, which represents huge savings for higher education institutions. On the other hand, using these programs, students can verify and supplement theoretical knowledge. The programs offer enough opportunities to students to solve problems in independently various areas of mathematics.

Figure 4 shows the function $z = x^2 + y^2$, $x\epsilon(-2,4)$ and $y\epsilon(1,3)$ we get by using Octave and QtOctave.

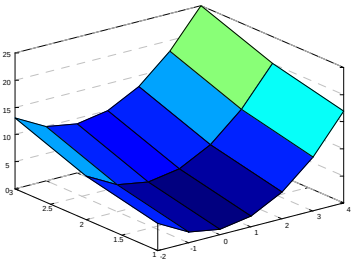

Figure 4. The function $z = x^2 + y^2$, $x\epsilon(-2,4)$, $y\epsilon(1,3)$

Figure 5. shows on the graph $y = (2x) / (1 + x^2)$ we get by using Maxima.

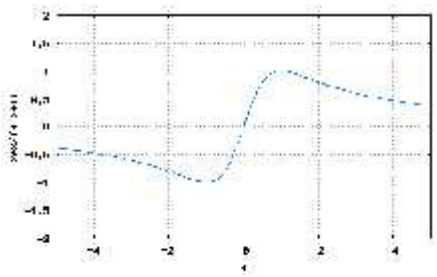

Figure 5. The graph $y = (2x) / (1 + x^2)$ we get by using Maxima.

CONCLUSION

In the process of learning it is important to let unfamiliar terms be explained by the available methods. In education, the most difficult thing is to adjust teaching content to student needs. It must be taken take care of comprehensibility, accessibility and quality of materials. Mathematical objects in virtual environments have always been a challenge for realization. Educational software offer great relief. Using them, basic concepts may be explained to students in a visual way, also, interactivity between professors and students may be increased so consequently and the quality of teaching develops. GeoGebra, Octave and Maxima are for free and easy to use. As such, they are not financial problem of higher education institutions. The problem may be a teachers desire and willingness to apply them. Given that the use of these programs help students to acknowledge mathematical subjects easily, that is why teachers must adapt. The strategy of educational institutions should be oriented towards it, therefore teaching content must be adjusted. These three tools are only a small contribution in a number of existing ones. As such they should be used maximum.

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